Number: ____________________

Name: ____________________

Teacher: ____________________

GOSFORD HIGH SCHOOL

HSC Mathematics 2012

Assessment Task 1

November 2011

TIME ALLOWED 65 MINUTES + 5 Minutes reading

- Show all working for questions worth more than 1 mark.
- An approved calculator may be used

Marks

	Functions	/20
	Calculus	/19
	Locus and the parabola	/17
	Total	/56

Question 1 Functions (20 Marks) **Marks**

a) Draw separate sketches of

(i) $y = |x + 2|$ **2**

(ii) $y = \frac{3}{x-2}$ **2**

(iii) $y = \sqrt{2 - x}$ **2**

(iv) $y = 2^{-x}$ **2**

b) State the domain and range for each of these functions. **4**

c) A circle of radius 5 units has its centre at the point (–3,4).

(i) Find its equation **2**

(ii) Find the coordinates of the points where the curve cuts the y axis. **2**

d) Graph the region in which $y < x^2 - 2$ and $x^2 + y^2 \leq 4$ both hold simultaneously. **4**

Question 2 Calculus (19 Marks) Start a New Sheet of Paper

a) Evaluate $\lim_{x \to 4} \frac{x^2-16}{x-4}$ **2**

b) Differentiate from first principles $f(x) = 8 - 7x - 2x^2$ **3**

c) Find the equation of the normal to the curve $y = (2x + 1)^4$ at the point where $x = -1$. Give your answer in the general form. **3**

d) Differentiate:

(i) $\frac{x^2-2x+1}{x}$ **2**

(ii) $\frac{1}{3x^2} + x\sqrt{x}$ **2**

Question 2 (continued) **Marks**

e) Differentiate and give answer in simplest form:

(i) $y = \dfrac{3x-1}{(1-2x)^3}$ **3**

(ii) Hence state the x coordinate(s) of any point(s) where the tangent to the curve is parallel to the x axis **1**

Differentiate and give answer in simplest form

(iii) $f(x) = x\sqrt{2x-1}$ **3**

Question 3 (Locus and the Parabola) (17 Marks) Start a New Sheet of Paper

a) For the parabola $x^2 = 12y$ write down the coordinates of the focus and the equation of the directrix. **2**

b) (i) Derive the locus of a point P which moves such that AP is perpendicular to PB if A is (3, 0) and B is (–1,6). **3**

(ii) Show that the locus is a circle and find its centre and radius. **2**

c) For the parabola $y^2 = 8(3-x)$ find: **4**

(i) The coordinates of the vertex

(ii) The coordinates of the focus

(iii) The equation of the directrix

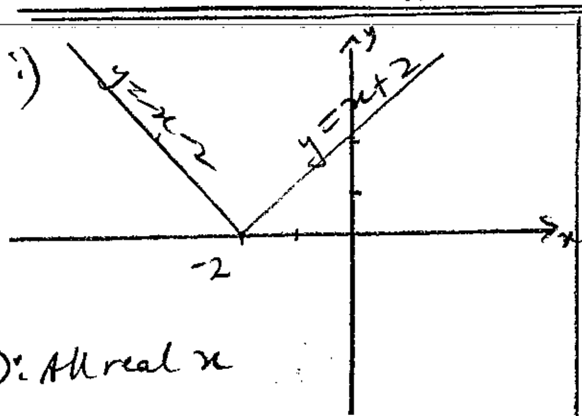
(iv) Sketch the parabola

d) (i) Derive the equation of the locus of a point P which is equidistant from the x axis and the line $4x + 3y = 12$. **3**

(ii) Describe geometrically this locus and draw a sketch of it together with the original lines. Label each graph. **3**

Solutions to HSC Task 1 2012

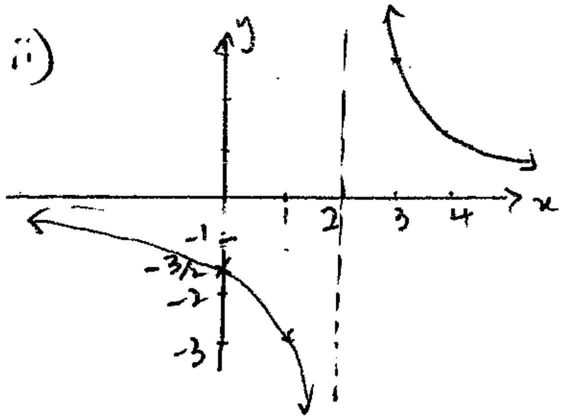
1 a) i)



D: All real x

R: $y \geq 0$

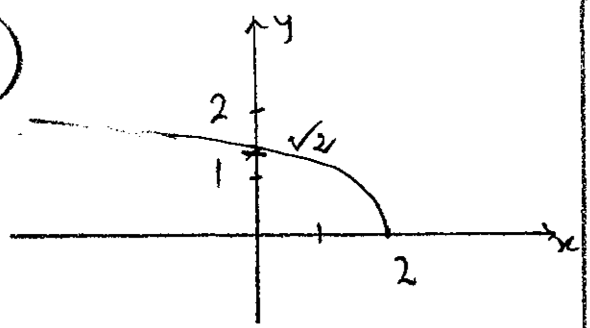
ii)



D: All real x except $x = 2$

R: All real y except $y = 0$

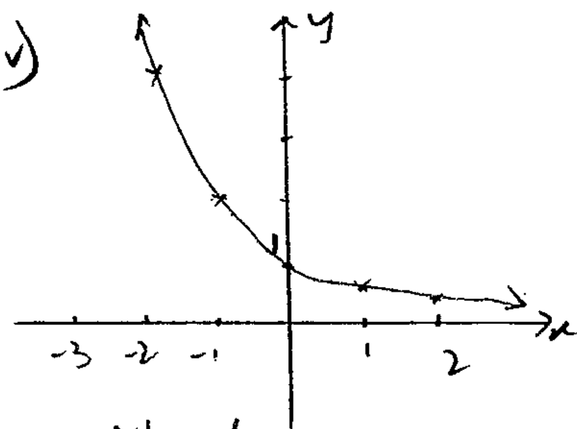
iii)



D: $x \leq 2$

R: $y \geq 0$

iv)



D: All real x

R: $y > 0$

c) i) $(x+3)^2 + (y-4)^2 = 25$

ii) y axis put $x = 0$

$$9 + (y-4)^2 = 25$$

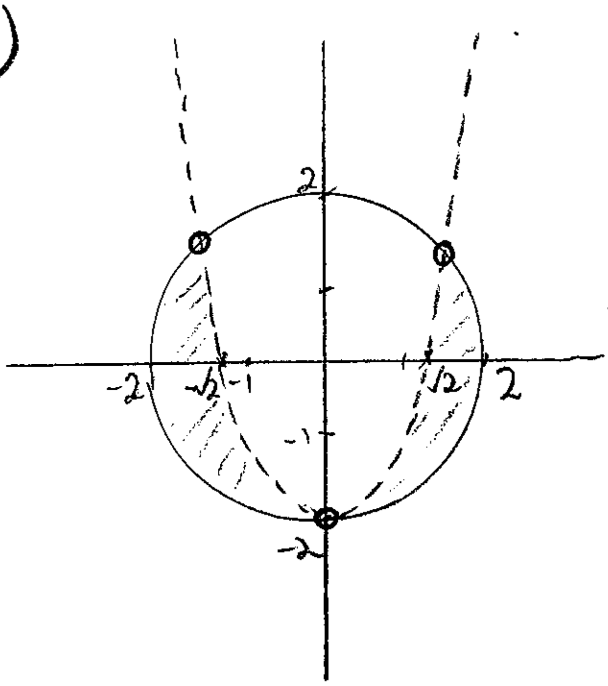
$$(y-4)^2 = 16$$

$$y - 4 = \pm 4$$

$$y = 8 \text{ or } 0$$

$\therefore$ Points are $(0,0)$ + $(0,8)$

d)



Test $(0,0)$

$0^2 + 0^2 \leq 4$ ✓

$\therefore$ Inside circle

$0 < 0 - 2$. False

$\therefore$ Outside parabola

2 a) $\lim_{x\to 4} \frac{(x-4)(x+4)}{x-4}$

$= \lim_{x\to 4} x+4 = 8$

b) $f(x) = 8-7x-2x^2$

$f(x+h) = 8-7(x+h)-2(x+h)^2$

$= 8-7x-7h-2x^2-4xh-2h^2$

$f(x+h)-f(x) = -7h-4xh-2h^2$

$\frac{f(x+h)-f(x)}{h} = -7-4x-2h$

$f'(x) = \lim_{h\to 0} -7-4x-2h$

$= -7-4x$

c) $y = (2x+1)^4$

$x=-1, \quad y = (-1)^4 = 1$

$y' = 4(2x+1)^3 . 2$

$= 8(2x+1)^3$

$x=-1, \quad m_1 = 8 \times (-1)^3$

$= -8$

$\therefore$ For normal

$m_2 = \frac{1}{8}$

$\therefore y-1 = \frac{1}{8}(x+1)$

$8y-8 = x+1$

$x-8y+9=0$

d) i) $y = x-2+x^{-1}$

$y' = 1-x^{-2}$

$= 1-\frac{1}{x^2}$ or $\frac{x^2-1}{x^2}$

ii) $y = \frac{1}{3}x^{-2} + x^{3/2}$

$y' = -\frac{2}{3}x^{-3} + \frac{3}{2}x^{\frac{1}{2}}$

$y' = \frac{-2}{3x^3} + \frac{3\sqrt{x}}{2}$

e) i) $y' = \frac{(1-2x)^3 . 3 - (3x-1) . 3(1-2x)^2 . -2}{(1-2x)^6}$

$y' = \frac{3(1-2x)^2 \{1-2x+2(3x-1)\}}{(1-2x)^6}$

$y' = \frac{3(4x-1)}{(1-2x)^4}$

ii) $y'=0$ when $x=\frac{1}{4}$

iii) $f(x) = x(2x-1)^{1/2}$

$f'(x) = x.\frac{1}{2}(2x-1)^{-1/2}.2 + (2x-1)^{1/2}.1$

$= \frac{x}{\sqrt{2x-1}} + \sqrt{2x-1}$

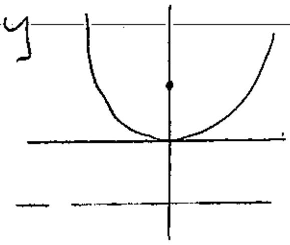
$= \frac{x+2x-1}{\sqrt{2x-1}}$

$= \frac{3x-1}{\sqrt{2x-1}}$

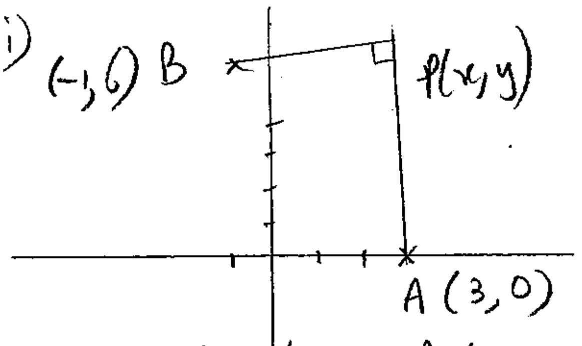
3. a) $x^2 = 4\times 3\times y$

S is $(0, 3)$

D is $y = -3$



b) i)



Use gradients or Pythagoras.

$m_1 = \frac{y}{x-3}$, $m_2 = \frac{y-6}{x+1}$

$\frac{y}{x-3}\cdot\frac{y-6}{x+1} = -1$

$y(y-6) = -(x-3)(x+1)$

$(x-3)(x+1) + y(y-6) = 0$

ii) $x^2 - 2x - 3 + y^2 - 6y = 0$

$x^2 - 2x + 1 + y^2 - 6y + 9 = 13$

$(x-1)^2 + (y-3)^2 = 13$

This is in the form

$(x-x_1)^2 + (y-y_1)^2 = r^2$

which is a circle

Centre $(1, 3)$ radius $\sqrt{13}$

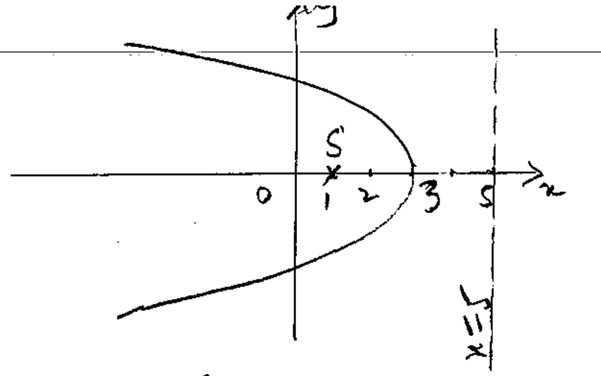
c) $y^2 = -8(x-3)$

$(y-0)^2 = -4\times 2(x-3)$

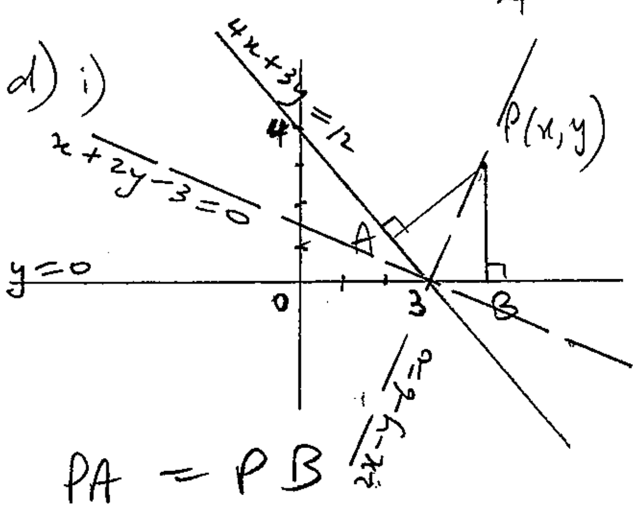
i) Vertex is $(3, 0)$

ii) Focus is $(1, 0)$

iii) Directrix is $x = 5$



d) i)



$PA = PB$

$\left|\frac{4x+3y-12}{\sqrt{4^2+3^2}}\right| = |y|$

$\therefore 4x+3y-12 = 5y$

or

$4x+3y-12 = -5y$

Equations are

$4x-2y-12 = 0$

or $2x-y-6 = 0$

and

$4x+8y-12 = 0$

or $x+2y-3 = 0$

ii) the locus is a pair of perpendicular straight lines, bisecting the angles between the x axis + $4x+3y=12$